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THE FEATURES OF MODELING OF THE FRIABLE MATERIAL MOVEMENT ALONG THE SPATIALLY VIBRATING SURFACE OF THE VIBRATORY MACHINE WORKING MEMBER

A movement of the technologic load (TL) in the form of the friable material along the spatially vibrating plane is considered. Dynamical and mathematical models of spatial movement of the three-mass vibratory systems — analogues of the vibratory technologic machines on the base of the systemic approach are developed. The obtained nonlinear interconnected differential equations can be used to control the movement of the TL by variation of the system dynamical and kinematical parameters. Some results of the numerical experiments showing a dependence of the vibratory displacement velocity on amplitude and frequency characteristics of the separate or combined spatial vibrations of the vibratory machine working member (WM) are given.

Keywords: *spatial vibrations of the vibratory machine working member, intensification of the material vibratory displacement, modeling of the movement*

Introduction. Many works are devoted to research into behavior of the friable materials under action of vibrations [1—9], but not all the aspects of the problem are studied sufficiently. The vibratory transportation and technologic machines and the processes related to the friable materials are especially diverse and widely used. The efficacy of these machines is stipulated by the numerous constructional and dynamical factors.

In the above mentioned machines nonworking spatial vibrations are often generated in the vibratory technologic machines disturbing a normal mode of their work. In this connection a problem of development of the spatial dynamical and corresponding mathematical models of these machines arises with the purpose of research into influence of the nonworking (parasitic) spatial vibrations on the proceeding technologic process.

The nonlinear interconnected equations of movement of the loaded vibratory machine allow us to determine influence of the separate as well as combined various vibrations on behavior of the technologic load. As a result, correlations of the system parameters favoring improvement of the quality and quantity of the produce to be processed (transportation, mixing, dosing, feeding of the friable material, etc.) can be revealed.

Building a dynamical model. A vibratory technologic machine can be considered as a three mass vibratory system including the elements: active mass (working member) — M_1, reactive mass (vibrating exciter) — M_2, material to be processed or transported — M_3 (figures 1, 2).

The main difference between the considered system and classical n mass spatial system consists of the following:
- specificity of the technologic mass M_3 (friable or piece material, etc.), performing a relative movement with respect to the mass of the working member M_1; at that the masses M_1 and M_2 perform independent movement under action of the external source of energy and mass M_3 — under action of the mass M_1;
- a certain initial location of masses M_1, M_2 and M_3 relative to each other (such condition makes asymmetric a common succession of building a mathematical model of the system movement);
- the features of interaction of masses M_1 and M_3 as connected with each other by the conventional elastic unilateral connection (figures 1—3).

To facilitate deduction of the equations we present a vibratory machine (figure 1) in the form of the classical three-mass vibratory system (figure 2) considering the above mentioned distinctive features.

Consider a field of forces acting on the system.

For obtaining a general vector and then analytical expression of the kinetic energy we determine absolute velocity of any material point of each mass A_i, B_i, C_i. They are connected by vectors R_j, R_{ji}, r_{ji}; $j = 1, 2, 3$ with the origins of the proper coordinate axes as well as with the origin of the inertial coordinate system $O\xi\varepsilon\zeta$. Besides, M_3 is connected with the center of gravity of mass M_1 (origin of the coordinate system $O_1x_1y_1z_1$). This connection can be unilateral depending on the technologic load and mode of its movement.

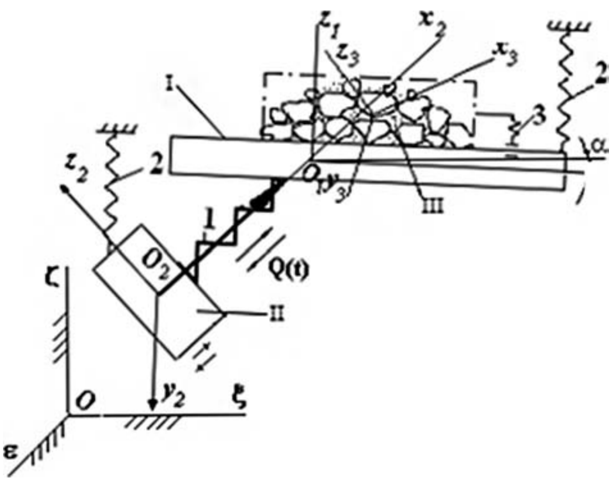

Figure 1 — Vibratory machine: I — working member; II — vibro-exciter; III — load

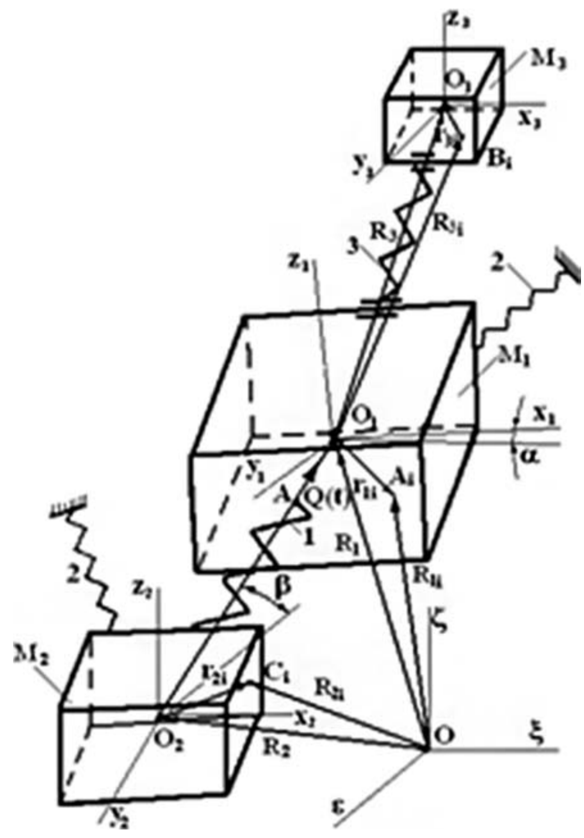

Figure 2 — The three mass vibratory system — analogue of the vibratory machine

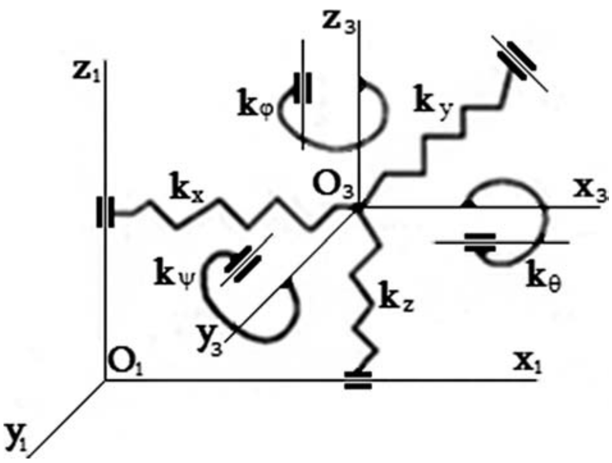

Figure 3 — Model of the friable material

Velocities of points A_i, B_i, and C_i have the vector forms

$$\vec{V}_{Ai} = \vec{V}_{O_1} + \omega_{O_1} \times \vec{r}_{1i};\ \vec{V}_{Ci} = \vec{V}_{O_2} + \omega_{O_2} \times \vec{r}_{2i};$$
$$\vec{V}_{Bi} = \vec{V}_{O_1} + \omega_{O_1} \times \vec{R}_{3i} + \vec{V}_{O_3} + \omega_{O_3} \times \vec{r}_{3i}. \quad (1)$$

Correspondingly, expressions for kinetic energy of masses will have the form

$$T_j = \frac{1}{2}\sum_{i=1}^{(n)_j}(M_j)_i[V_{Ai,Bi,Ci}]^2, \quad (2)$$

where M_{1i}, M_{2i}, M_{3i} are masses of the particles A_i, B_i, C_i; n_1, n_2, n_3 — number of corresponding particles;

ω_{O1}, ω_{O2}, ω_{O3} — angular velocities of the corresponding particles relative to the mass centers.

The rotary movements of masses M_1, M_2, M_3 will be described by directing cosines of the ship angles of Euler [10] (ensuring little changes of Euler's angles at little deviations of masses) considering the angles of inclinations of the working member vibrating surface (α) and directions of the exciting force (β).

For obtaining analytical expressions of movement of the mentioned interconnected vibratory system in space it is necessary to determine coordinates of the mass centers, points of fastening of the elastic elements to the masses and reduce them to one system of coordinates. Since technologic load M_3 is directly connected with the working member M_1 it is expedient to choose such system $O_1x_1y_1z_1$.

The projections of coordinates of points will be determined with the help of directing cosines of the angles between axes of the systems of coordinates Oxy and $O_1'x_1'y_1'z_1'$. As an example we will bring expressions of coordinates of the fastening of point A of the main elastic system 1 (figure 2) in the coordinate system $O_1x_1y_1z_1$ after dynamical displacement of masses:

$$x_A = x_{O_1} + x_{1A}\nu_{11} + y_{1A}\nu_{12} + z_{1A}\nu_{13};$$

$$y_A = y_{O_1} + x_{1A}\nu_{21} + y_{1A}\nu_{22} + z_{1A}\nu_{23}; \ldots; \ z_A,$$

where x_{O_1}, y_{O_1}, z_{O_1} are coordinates of point O_1 (after displacement); x_{1A}, y_{1A}, z_{1A} — coordinates of point A; ν_{11}, ..., ν_{23}, ..., ν_{33} — directing cosines of the angles between axes of the coordinate systems $O_1x_1y_1z_1$ and $O_1'x_1'y_1'z_1'$, i. e. between initial and dynamical locations of the mass M_1 (figure 2).

For the illustration we give an analytical expression of the kinetic energy of mass M_1:

$$\begin{aligned} T_1 = \frac{1}{2}M_1(\dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2) + \frac{1}{2}[(J_{x_1}\cos^2\alpha_1 + \\ + J_{z_1}\sin^2\alpha_1)\dot{\theta}_1^2 + (J_{x_1}\sin^2\alpha_1 + J_{z_1}\cos^2\alpha_1)\dot{\varphi}_1^2] + \\ + J_{y_1}\dot{\psi}_1^2 + \dot{\theta}_1\dot{\varphi}_1(J_{x_1} - J_{z_1})\sin\alpha_1\cos\alpha_1, \end{aligned} \quad (3)$$

where J_{x_1}, J_{y_1}, J_{z_1} are moments of inertia of mass M_1 about axes of the system $O_1x_1y_1z_1$; x_1, y_1, z_1, q_1, y_1, j_1 — coordinates of spatial movements of the mass M_1.

Expansion of the kinetic energy of mass M_3 has a more complicated form since it performs a relative movement with respect to moving mass M_1 (we did not consider it necessary to bring it here).

For inclusion of the TL (mass M_3) in the general spatial system (figure 2) and giving it generalized character we present it formally as a rigid body connected to the WM (mass M_1) by the conventional elastic system 3 (figure 3) describing elastic features of the friable material [3, 11].

At the fixed moment of time elastic system 3 (as well as 1 and 2 figures 1, 2) is decomposed into three components describing elastic properties of the material in the space.

The elastic system 3 is featured by the non-holding character of its connection with the WM in dynamics. With the help of the elastic and damping elements internal layer characteristics of the techno-logic friable material, interaction between layers and between lower layer and the WM are described. It differs from the existing models [1, 2] by the fact that it considers all degrees of freedom, i. e. it can be included in the model of the general spatial system (figures 1, 2) and depending on the concrete problem reduced to the simpler form (plane, linear, dotted).

Presentation of the TL by the rigid body (at deduction of expressions for kinetic energy) is stipulated by the necessity of obtaining an equation of movement in the more generalized form not only for translational (in this case TL would be considered as a material point) but for rotary movements also.

Deformation of the friable TL layer is modeled by the elastic elements with the coefficients of elasticity $k_{x3}, k_{y3}, k_{z3}, k_{\theta3}, k_{\psi3}, k_{\varphi3}$ (figure 3). Dissipation of energy at deformation of the layer is taken into account by the dampers with the coefficients of resistance $c_{x3}, c_{y3}, c_{z3}, c_{\theta3}, c_{\psi3}, c_{\varphi3}$ (not shown in the figure). Thus, direct contact of the TL and WM is replaced by the elastic and frictional connections.

The two approaches can be used to determine the potential forces of elastic systems of the WM and TL depending on the displacement value. In the first case (at small displacements) the components of the elastic forces along the coordinate axes are determined according to potential energy and equations of Lagrange. In the second case (at great, for example resonant displacements) a change in the length of the elastic system 1 is determined and it is decomposed into components along the coordinate axes considering the coefficient of rigidity. The dimensions of the conventional elastic system 3 can be determined approximately depending on the location of the TL relative to the concrete surfaces of the WM.

Deduction of the differential equations. For deduction of the equations of spatial movement of the three-mass vibratory system (figures 1, 2) equation of Lagrange of the second order is used in the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} = Q_q + Q_q', \quad (4)$$

where T is the sum of the kinetic energy of masses M_1, M_2, M_3 drawn up similarly to (3) for each mass; q — generalized coordinate taking the values x_1, y_1, z_1,..., y_3, z_3, θ_1, ψ_1, φ_1,...,ψ_3, φ_3; Q_q' — potential (elastic) forces and moments stipulated by the machine elastic system; Q_q — forces not related to deformations of the elastic system or inertness of the vibratory system under consideration: external (exciting) forces; weights of gravity; forces of resistance of the type of external friction (friction force between TL and WM).

On the base of adopted assumptions about smallness of the rotary displacements the derivatives of variables not greater than second order will be considered in the equations.

Since interaction of masses M_1 and M_3 is more specific among the interconnected masses and M_3 can have various internal structures, we will bring here equations of movement of these masses along the coordinate axis x (a method of deduction of the equations of movement of all masses along other coordinates is similar and we do not bring them here). For mass M_1 we have:

$$\begin{gathered}(M_1+M_3)\ddot{x}_1+M_3[(\ddot{\psi}_1 z_3+2\dot{\psi}_1\dot{z}_3-\\ -\ddot{\varphi}_1 y_3-2\dot{\varphi}_1\dot{y}_3-\ddot{y}_3\varphi_1+\ddot{z}_3\psi_1)\cos\alpha_1+\\ +\ddot{x}_3\cos\alpha_1+\ddot{z}_3\cos\alpha_1+(\ddot{\theta}_1 y_3-2\dot{\theta}_1\dot{y}_3-\\ -\ddot{\psi}_1 x_3-2\dot{x}_3\dot{\psi}_1+\ddot{y}_3\theta_1+\ddot{x}_3\psi_1)\sin\alpha_1]=\\ =Q(t)(\psi_1\sin\alpha_1+\cos\alpha_1)+\\ +f(k_q,q,q_iq_j,c_q,\dot{q},\dot{q}_i\dot{q}_j),\end{gathered} \tag{5}$$

where $a_1 = a + b$.

For mass M_3:

$$\begin{gathered}M_3[(\ddot{x}_3+\ddot{x}_1-\ddot{z}_1\psi_1)\cos\alpha_1-(\ddot{z}_1+\ddot{x}_1\psi_1)\sin\alpha_1-\\ -(2\dot{\psi}_1\dot{z}_3-\ddot{\varphi}_1 y_3+2\dot{\varphi}_1\dot{y}_3-\ddot{y}_1\varphi_1-\ddot{\psi}_3 z_3)]=\\ =f_xN_z sign(\dot{x}_3)+f^*(k_q^*,q,q_iq_j,c_q^*,\dot{q},\dot{q}_i\dot{q}_j),\end{gathered} \tag{6}$$

where f, f^* are the functions of the coordinates, velocities and characteristics (k_q, c_q and k^*_q, c^*_q) of the elastic system and their derivatives; $Q(t)$ — exciting force of the vibro-exciter; "*sign*" — nonlinear function depending on the sign of velocity

$$\dot{x}_3,\dot{y}_3,\dot{z}_3:\ sign=1 \text{ at } \dot{x}_3(\dot{y}_3,\dot{z}_3)<0;$$

$$sign=-1 \text{ at } \dot{x}_3(\dot{y}_3,\dot{z}_3)>0;$$

N_z — reaction of the TL on the WM; f_x — friction coefficient between the material and surface of the WM.

Equations (5), (6) and remaining ones, which are not given here, describe movements of the three-mass system (figures 1, 2) and they are interconnected by the nonlinear summands of potential and inertial character, while the form of connection for both systems are similar. The only difference is in presence of the sum of masses M_1 and M_2 in equations (5) while only one mass M_3 is present in equations (6).

It should be noted that equations (5) and (6) describe movement of the mass M_3 relative to M_1 at constant interconnection (at constant contact), and a potential field in the form of Q_q (in the right-hand part) is an elastic and damping characteristic of mass M_3 (in the case of the friable material) and it can vary depending on its location Q_q (and coefficients k_q^*, c_q^* also). Besides, dynamical dependence of M_1 and M_3 can be stepwise. In this case conditions of tossing of the material M_3 on the vibrating surface M_1 are added to the systems (5) and (6).

Solution of the differential equations. Some results of solution of the equations (5) and (6) are given in figures 4, 5 and 6. Namely, dependences of the velocity of displacement of the friable material V_x on the amplitude of the rotary (ψ and φ) and vertical (z_1) vibrations of the WM are shown. In each case the

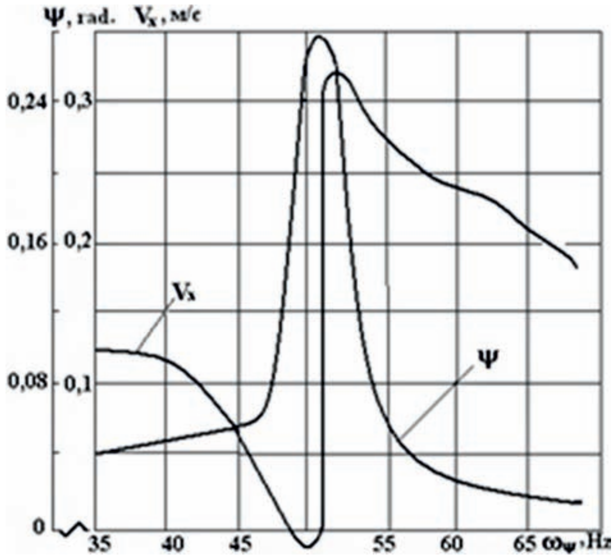

Figure 4 — Dependence of the material velocity on the rotary vibrations ψ of the WM

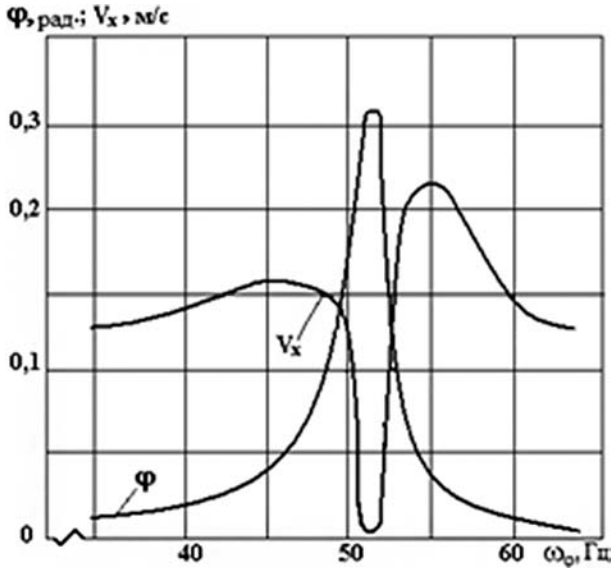

Figure 5 — Dependence of the material velocity on the rotary vibrations φ of the WM

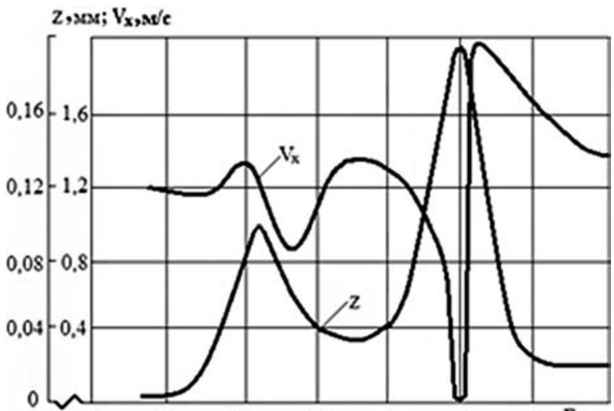

Figure 6 — Dependence of the material velocity on the vertical vibrations of the WM

resonances were provoked in the indicated directions (in the figure 6 two resonant peaks: sub- and main peaks are shown). As it is seen from these dependences the velocity varies significantly in the range of the resonance that indicates possibility of use of the given fact for intensification of the vibratory displacement of the friable material.

Conclusions. The given spatial dynamical and mathematical models of the system "vibro-exciter — vibrating plane — friable material" allow studying behavior of the friable material at change of the dynamical, physical and geometric parameters of the system. With the help of the presented mathematical model influence of the non-working spatial vibrations of the WM and other parameters of the vibro-machine on the technologic process can be studied. This will allow increasing a degree of its purposeful application (for example, combination of some spatial vibrations with working vibrations increases intensity of the technologic process).

Later on, research and development of the constructions with the use of a combination of the working vibrations and some spatial vibrations ensuring improvement of the technologic process (for example, increase in the vibratory displacement velocity) are planned.

References

1. Blekhman I.I. *Vibraciya v technike* [Vibrations in Technic]. Moscow, 1981, Nauka, vol. 4. 460 p.
2. Goncharevich I.F. *Dinamika vibracionnogo transportirovaniya* [Dynamics of vibratory transportations]. Moscow, Nauka, 1978. 224 p.
3. Zviadauri V.S. *Dinamika vibracionniyh transportno-technologicheskiyh mashin* [Dynamics of the vibratory transport-technological machines]. Tbilisi, Mecniereba, 2001. 192 p.
4. Sloot E.M., Kruyt N.P. Theoretical and experimental study of the transport of granular materials by inclined vibratory conveyors. *Powder Technology*, 1995, pp. 203–210.
5. Saeki M. Takano E. Vibratory conveyance of granular materials. *Proc. 5th Int. Con. on sound and vibration*. South Australia, Adelaide, 1997. 7 p.
6. Despotovich Z., Sinik V., Jankovich S., Dobrilovic D., Bjelica M. Some specific of vibratory conveyor drives. *Proc. V Int. Conf. "Industrial Engineering and Environmental Protection"*. Serbia, Zrenjanin, 2015, pp. 247–253.
7. Vaisberg L.A., Demidov I.V., Ivanov K.S. Mehanika sypuchih sred pri vibracionnyh vozdejstvijah: metody opisanija i matematicheskogo modelirovanija [Mechanics of the friable medium at the vibratory impact: Methods of description and mathematical modeling]. *Obogashchenie rud* [Ore benefication], 2015, no. 4, pp. 21–31.
8. Sorokin S.A. Vlijanie amplitudnochastotnyh harakteristik vibracionnogo dozatora na ravnomernost istechenija smesi [Influence of the amplitude and frequency characteristics of the vibratory dozer on the regularity of the mixture outflow]. *Selskoe, lesnoe i vodnoe hozjajstvo* [Agriculture, forestry and water management], 2012, no. 5. 6 p.
9. Tai C.H., Hsiau S.S. Dynamics behaviors of powders in a vibrating bed. *Powder Technology*, 2004, 139, pp. 221–132.
10. Krilov A.N. *Kachka korablya* [Roll of the ship]. Moscow, Nauka, 1971, vol. 9. 456 p.
11. Ganiev R.F., Kononenko V.O. *Kolebaniya tviordiyh tel* [Oscillations of solid bodies]. Moscow, Nauka, 1976. 433 p.
12. Cherno A.A., Gurov A.P. Optimizacija chastoty kolebanij vibracionnyh ustanovok s jelektromagnitnym privodom po kriteriju maksimuma KPD [Optimization of the frequency of vibrations of the vibratory devices with electromagnetic drives on the criterion of maximum efficiency]. *Vestnik NTU "HPI"* [Bulletin of NTU "KhPI"], 2015, no. 12(1121), pp. 360–363.

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ОСОБЕННОСТИ МОДЕЛИРОВАНИЯ ДВИЖЕНИЯ СЫПУЧЕГО МАТЕРИАЛА ПО ПРОСТРАНСТВЕННО-ВИБРИРУЮЩЕЙ ПОВЕРХНОСТИ РАБОЧЕГО ОРГАНА ВИБРОМАШИНЫ

Рассмотрено движение технологической нагрузки (ТН) в виде сыпучего материала на пространственно-вибрирующей плоскости. С этой целью, на основе системного подхода, разработаны динамическая и математическая модели пространственного движения трехмассовой колебательной системы — аналога вибрационной технологической машины. Дифференциальные уравнения являются нелинейно взаимосвязанными и с их помощью можно управлять движением ТН в зависимости от изменения динамических и кинематических параметров всей системы. Приведены некоторые результаты численного эксперимента, показывающие зависимость скорости вибрационного перемещения от амплитудно-частотных характеристик отдельных или комбинационных пространственных колебаний рабочего органа вибромашины.

Ключевые слова: *пространственные колебания рабочего органа вибромашины, интенсификация вибрационного перемещения материала, моделирование движения*

Список литературы

1. Вибрации в технике: в 6 т. / ред. совет: В.Н. Челомей (пред.). — М.: Наука, 1981. — Т. 4. Вибрационные процессы и машины / под ред. Э.Э. Лавендела. — 468 с.
2. Гончаревич, И.Ф. Динамика вибрационного транспортирования / И.Ф. Гончаревич. — М.: Наука, 1978. — 224 с.
3. Звиадаури, В.С. Динамика вибрационных транспортно-технологических машин / В.С. Звиадаури. — Тбилиси: Мецниереба, 2001. — 192 с.
4. Sloot, E.M. Theoretical and experimental study of the transport of granular materials by inclined vibratory conveyors / E.M. Sloot, N.P. Kruyt // Powder Technology. — 1995. — Pp. 203–210.
5. Saeki, M. Vibratory conveyance of granular materials / M. Saeki, E. Takano // Fifth international congress on sound and vibration, Adelaide, South Australia. 1997. — 7 p.
6. Some specific of vibratory conveyor drives / Z. Despotovich [et al.] // V International Conference Industrial Engineering and Environmental protection, Zrenjanin, Serbia 2015. — Pp. 247–253.
7. Вайсберг, Л.А. Механика сыпучих сред при вибрационных воздействиях: методы описания и математического моделирования / Л.А. Вайсберг, И.В. Демидов, К.С. Иванов // Обогащение руд. — 2015. — № 4. — С. 21–31.
8. Сорокин, С.А. Влияние амплитудно-частотных характеристик вибрационного дозатора на равномерность истечения смеси / С.А. Сорокин // Сельское, лесное и водное хозяйство. — 2012. — № 5. — 6 с.
9. Tai, C.H. Dynamics behaviors of powders in a vibrating bed / C.H. Tai, S.S. Hsiau // Powder Technology. — 2004. — № 139. — Pp. 221–132.
10. Крылов, А.Н. Качка корабля: в 12 т. / А.Н. Крылов. — М.: Наука, 1971. — Т. 9. — 456 с.
11. Ганиев, Р.Ф. Колебания твердых тел. / Р.Ф. Ганиев, В.О. Кононенко. — М.: Наука, 1976. — 433 с.
12. Черно, А.А. Оптимизация частоты колебаний вибрационных установок с электромагнитным приводом по критерию максимума КПД / А.А. Черно, А.П. Гуров // Вестник НТУ «ХПИ». — 2015. — № 12(1121). — С. 360–363.