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METHODS FOR CALCULATING THE LOAD OF ELECTRIC VEHICLE GEARBOXES USING THEIR DYNAMIC MODELS

A two-mass dynamic model of an electric vehicle is considered to obtain analytical dependences describing the gearshift process in a shaft-planetary gearbox. The calculations are based on the optimal diagram of shifting gears from low to high without interruption of the power flow from the electric motor. At the stage of gear shifting, at which the kinematic gear ratio of the gearbox changes from u_1 to u_2, Lagrange equations of the second kind are used to determine the time dependences of angular velocities of the input and output shafts of gearbox. An example is given for calculating the gearshift time from low gear to high one for the MAZ-4381EE delivery electric truck, as well as a refined calculation for a multi-mass dynamic scheme is carried out.

Keywords: *electric truck, gearbox, electric motor, diagram of shifting gears, power circulation, dynamic model, friction clutch, Lagrange equations of the second kind, slipping time*

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Introduction. To date, a significant number of electric vehicles produced either do not contain gearboxes or contain single-speed gearboxes. The use of multi-speed gearboxes in power units can significantly increase the energy efficiency of electric vehicles [1–3].

In urban conditions, the movement of an electric vehicle at a constant speed is 15–25 % of the operating time. Accelerated movement (acceleration) accounts for 30–45 % of the time [4]. In this paper, the process of shifting gears from low to high in a two-speed gearbox is investigated during acceleration of the electric vehicle. A typical traction characteristic of such a vehicle is shown in Figure 1.

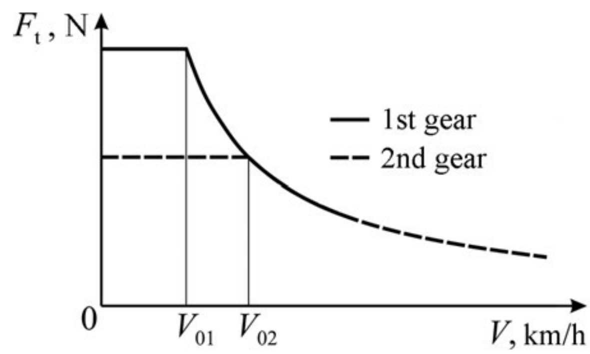

Figure 1 — Traction characteristics of an electric vehicle with two-speed gearbox

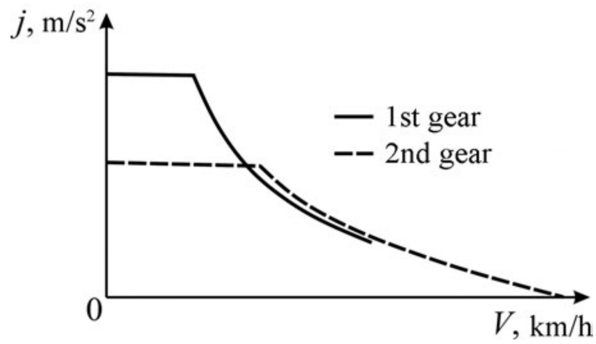

Figure 2 — Acceleration graph of the electric vehicle with two-speed gearbox

The curved part of the traction characteristic is described by a hyperbolic function and is a section of constant power. In Figure 1, the values of the speeds V_{01}, V_{02} correspond to the nominal values of the traction forces (at the rated power of the electric motor) on the 1st and 2nd gears.

Figure 2 shows the acceleration graph of an electric vehicle corresponding to the traction characteristic shown above.

Graphs of acceleration time and starting distance make it possible to estimate the acceleration capability of the designed electric vehicle, as well as, taking into account the efficiency maps of the electric motor, to calculate the energy consumed by traction batteries.

The article [5] presents analytical and numerical methods for calculating the traction and speed characteristics of mobile machines with electromechanical power units containing traction motors with hyperbolic mechanical characteristic and single-speed gearboxes.

When calculating the acceleration time and starting distance of electric vehicles with multi-speed gearboxes, it is necessary to take into account the gearshift time, which significantly depends on the shifting process itself in the power unit [6–9]. The gearshift process can be carried out both with an interruption of the power flow and without an interruption.

Analytical calculation method for a two-mass dynamic model. In modern automated gearboxes, gear shifting occurs without interruption of the power flow. In this paper, the gear shifting process from low to high is considered by the example of a shaft-planetary gearbox scheme. Figure 3 shows the two-mass dynamic model of an electric vehicle power unit with such a gearbox.

The assumed simplified dynamic model of the car does not take into account the elastic properties of the transmission.

The process of gear shifting with optimal overlap can be represented in the form of a diagram (Figure 4). At the same time, it is assumed that there is no power circulation in the circuit of the shifted gears, shifting occurs with minimal slipping losses in the friction clutch and minimal dynamic loads in the transmission.

The gear shifting process is divided into two stages. The first stage continues from the time of the start of engagement $t = 0$ (see Figure 4) of the friction clutch FC_2 to the moment $t = t_1$, at which the torque transmitted by it

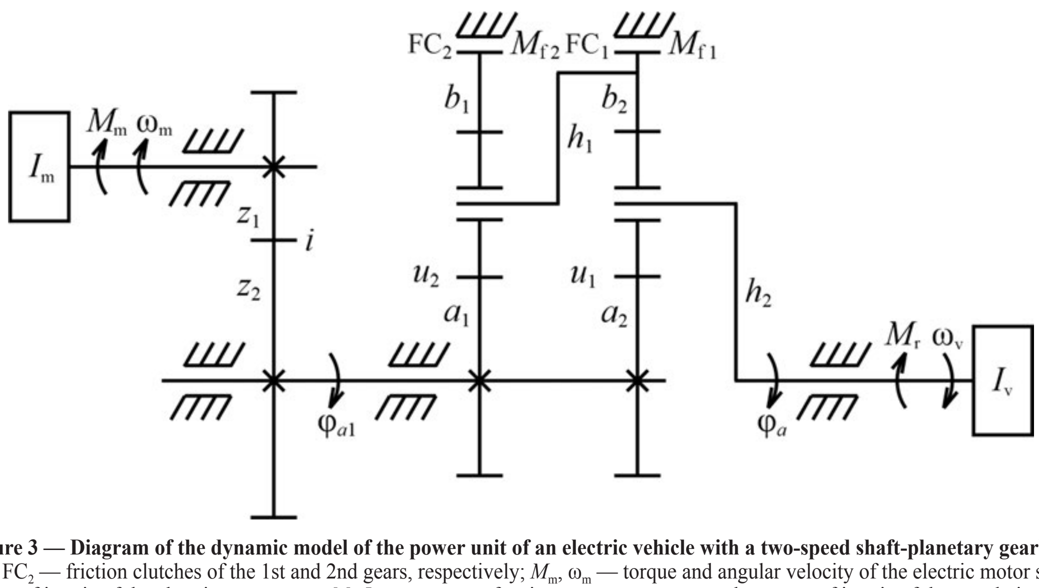

Figure 3 — Diagram of the dynamic model of the power unit of an electric vehicle with a two-speed shaft-planetary gearbox:
FC_1, FC_2 — friction clutches of the 1st and 2nd gears, respectively; M_m, ω_m — torque and angular velocity of the electric motor shaft; I_m — moment of inertia of the electric motor rotor; M_r, I_v — moment of resistance to movement and moment of inertia of the translationally moving mass and wheels of the electric vehicle, reduced to the output shaft of the gearbox; ω_v — angular velocity of the output shaft of the gearbox; u_1, u_2 — gear ratios of the gearbox in 1st and 2nd gears, respectively; i — gear ratio of single reduction gear z_1–z_2 with fixed axles

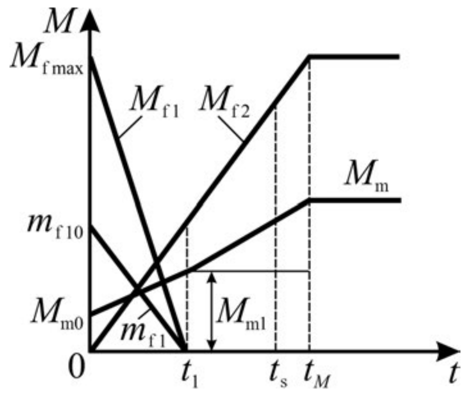

Figure 4 — Optimal diagram of the gear shift of the electric vehicle gearbox from the low u_1 to the high u_2 without interrupting the power flow from the electric motor

reaches the value necessary to overcome the moment of resistance M_r.

The moment of friction $M_{f2}(t)$ in the friction clutch FC_2 is assumed to increase in proportion to the time t:

$$M_{f2}(t) = \begin{cases} k_\mu t & \text{with} \quad 0 < t < t_M, \\ M_{f\max} & \text{with} \quad t \geq t_M, \end{cases} \tag{1}$$

where k_μ = const is the rate of the friction clutch engagement; $t_M = M_{f\max}/k_\mu$ is the time of increase of the moment of friction from zero to the maximum value; $M_{f\max}$ is the maximum moment of friction in the friction clutches, assumed to be the same for FC_1 and FC_2 and determined by the most loaded clutch FC_1 when working in first gear by equation

$$M_{f\max} = M_{EMn}\beta i k_2 \eta_i \eta_1, \tag{2}$$

where M_{EMn} is the rated torque developed by the electric motor; β is the friction clutch safety factor with a static coefficient of friction; $k_2 = z_{b2}/z_{a2}$ is the kinematic parameter of the second planetary gear set of the gearbox; z_{b2}, z_{a2} are the numbers of teeth of the crown and sun gears of the second planetary gear set of the gearbox, respectively; η_i is the single-reduction gear efficiency; η_1 is the first gear efficiency.

The actual moment m_{f1} transmitted by the friction clutch FC_1 during this period of time decreases from the value of $m_{f10} = M_r k_2/u_1$ with $t = 0$ to zero with $t = t_1$ according to the linear law [6]. The engine torque increases from the value $M_{m0} = M_r/(iu_1\eta_i\eta_1)$ with $t = 0$ to the value $M_{m1} = M_r/(iu_2\eta_i\eta_1)$ with $t = t_1$ according to the linear law, too. When shifting gears from low to high, the disengagement rate of the friction clutch FC_1 is set so that in the time interval (t_0, ...) the actual moment of friction m_{f1} remains less than the limit value M_{f1} (see Figure 4).

At the end of the first stage, the torque of the electric motor corresponds to the operation of the gearbox in the 2nd gear, and the angular speed of rotation of the output shaft of the gearbox is in the 1st gear.

The second stage is associated with the disengagement of the friction clutch FC_1 ($M_{f1} = 0$) with $t = t_1$. At the same time, there will be no reduction in the electric vehicle speed, since the M_{f2} torque developed by the friction clutch is sufficient to overcome the resistance to movement when working in 2nd gear.

The value of the moment M_{f2} at $t = t_1$ is determined from the expression

$$M_{f2} = \frac{k_1 k_2}{1 + k_1 + k_2} \cdot \frac{M_r}{u_2},$$

where $k_1 = z_{b1}/z_{a1}$ is the kinematic parameter of the 1st planetary gear set of the gearbox.

The optimal overlap time t_1 is calculated by the formula

$$t_1 = \frac{M_{f2}}{k_\mu} = \frac{k_1 k_2}{1 + k_1 + k_2} \cdot \frac{M_r}{u_2 k_\mu}. \tag{3}$$

The moment M_r of the resistance to the electric vehicle movement, reduced to the output shaft of the gearbox, is determined from the expression

$$M_r = \frac{(F_{rd} + F_a) r_w}{u_{main}\eta_{main}}, \tag{4}$$

where F_{rd} is the road resistance force, N; F_a is the air resistance force, N; r_w is the car wheel radius, m; u_{main} is the gear ratio of the main gear; η_{main} is the main gear efficiency.

After substituting the expressions for F_{rd} and F_a in (4), we get [4]

$$M_r = \frac{\left[\left(f\cos\alpha + \sin\alpha\right) m_v g + k_a F_v V^2\right] r_w}{u_{main}\eta_{main}}, \tag{5}$$

where m_v is the gross vehicle weight, kg; g = 9.81 is the gravitational acceleration, m/s^2; f is the coefficient of rolling resistance; α is the angle of the road slope; k_a is the coefficient of air resistance, N·s^2/m^4; F_v is the frontal area of the vehicle, m^2; V is the vehicle speed, m/s.

After substituting (5) into (3), we get

$$t_1 = \frac{k_1 k_2}{1 + k_1 + k_2} \cdot \frac{[(f\cos\alpha + \sin\alpha) m_v g + k_a F_v V^2] r_w}{u_2 k_\mu u_{main}\eta_{main}}. \tag{6}$$

At the second stage of the gear shifting ($t \geq t_1$), the kinematic gear ratio of the gearbox changes from u_1 to u_2. This process ends at time t_s after the end of slipping of the friction clutch FC_2. The kinematic scheme of the process is shown in Figure 5.

At this stage, the system shown in Figure 3 has two degrees of freedom, the element position of which is determined by two independent generalized coordinates — the angles of rotation φ_{a1} of the input shaft and φ_a of the output shaft. The rotation angle φ_{b1} of the crown gear b_1 corresponding to these coordinates will be determined by the formula

$$\varphi_{b1} = \alpha_2\varphi_a - \alpha_1\varphi_{a1}, \tag{7}$$

where α_1, α_2 are dimensionless parameters of the speed gearbox,

$$\alpha_1 = \frac{1 + k_1 + k_2}{k_1 k_2}; \quad \alpha_2 = \frac{(1 + k_1)(1 + k_2)}{k_1 k_2}. \tag{8}$$

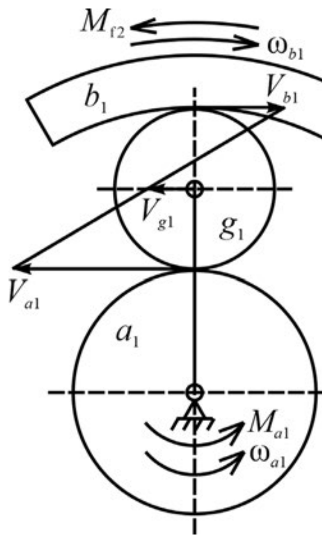

Figure 5 — Kinematic diagram of the slipping process of the friction clutch FC$_2$: ω_{b1} — angular velocity of slipping of the crown gear rim b_1 of the 1st planetary gear set of the gearbox; ω_{a1} — angular velocity of the sun gear a_1; V_{b1} — circumferential sliding speed of the crown gear relative to the clutch FC$_2$; V_{g1} — circumferential speed of the satellite axis g_1; V_{a1} — circumferential speed of the sun gear a_1; M_{a1} — torque on the input shaft of the gearbox

The angular velocities of the motor shaft and the input shaft of the gearbox are related by the ratio

$$\omega_m = i\omega_{a1}. \quad (9)$$

To determine the dependencies of angular velocities on time $\dot{\varphi}_{a1} = \omega_{a1}(t)$ and $\dot{\varphi}_a = \omega_a(t)$, let us use Lagrange equations of the second kind.

The Lagrange equations for generalized coordinates φ_{a1}, φ_a have the form

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\varphi}_{a1}} - \frac{\partial T}{\partial \varphi_{a1}} = Q_{\varphi_{a1}}; \quad \frac{d}{dt}\frac{\partial T}{\partial \dot{\varphi}_{a}} - \frac{\partial T}{\partial \varphi_{a}} = Q_{\varphi_{a}}, \quad (10)$$

where T is the kinetic energy of the system shown in Figure 3; $Q_{\varphi a1}$, $Q_{\varphi a}$ are generalized forces corresponding to generalized coordinates.

The kinetic energy of the system, taking into account (9), is determined by the expression

$$T = \frac{1}{2}(I_m i^2)\omega_{a1}^2 + \frac{1}{2}(I_a)\omega_a^2. \quad (11)$$

The following moments are applied to the system: M_m is the electric motor torque; M_r is the moment of resistance to movement; M_{f2} is the moment of friction between the crown gear b_1 and the friction clutch FC$_2$.

The generalized forces $Q_{\varphi a1}$, $Q_{\varphi a}$ are found by calculating the sums of the works of the above moments at the possible angular displacements $\delta\varphi_{a1}$, $\delta\varphi_a$

$$Q_{\varphi_{a1}} = M_m i - \alpha_1 M_{f2}; \quad Q_{\varphi_a} = \alpha_2 M_{f2} - M_r. \quad (12)$$

After calculating the derivatives in equations (10) of the system kinetic energy (11) and equating them to the corresponding generalized forces (12), two differential equations describing the process of changing the gear ratio of the gearbox are obtained:

$$I_m i^2 \frac{d\omega_{a1}}{dt} = M_m i - \alpha_1 M_{f2}; \quad I_v \frac{d\omega_a}{dt} = -M_r + \alpha_2 M_{f2}. \quad (13)$$

The integration of equations (13) is performed under the following initial conditions:

$$t = t_1 : \omega_{a1}(t_1) = \frac{V_1 u_{main} u_1}{r_w}; \quad \omega_a(t_1) = \frac{V_1 u_{main}}{r_w}, \quad (14)$$

where V_1 is the electric vehicle speed in the 1st gear at the beginning of the second stage.

The moment of friction $M_{f2}(t)$ in the friction clutch FC$_2$ is assumed to increase in proportion to the time t in accordance with expression (1). It is also assumed that the motor torque varies according to a linear law

$$M_m(t) = M_m(t_1) + k_m(t - t_1) = \frac{M_r}{i u_2 \eta_i \eta_2} + k_m(t - t_1), \quad (15)$$

where k_m is the rate of increase in the electric motor torque, N·m/s; η_2 is the efficiency of the 2nd gear.

After integrating the equations (13), taking into account the initial conditions (14), we get

$$\omega_{a1}(t) = \frac{V_1 u_{main} u_1}{r_w} + $$
$$+ \frac{1}{I_m i^2}\left[M_m(t_1) i (t - t_1) + k_m i \frac{(t - t_1)^2}{2} - \alpha_1 k_\mu \frac{t^2 - t_1^2}{2}\right]; \quad (16)$$
$$\omega_a(t) = \frac{V_1 u_{main}}{r_w} + \frac{1}{I_v}\left[-M_r(t - t_1) + \alpha_2 k_\mu \frac{t^2 - t_1^2}{2}\right].$$

The slipping time $t = t_s$ of the rim of the crown gear of the 1st gear set b_1 is found from the condition that its angular velocity of rotation is equal to zero $\omega_{b1} = 0$, which, after differentiation by time t of expression (7), leads to the equation

$$\alpha_2 \omega_a(t) - \alpha_1 \omega_{a1}(t) = 0. \quad (17)$$

The calculation of the slipping time t_s, during which the gear changes from the 1st to the 2nd, is performed for the MAZ-4381EE electric truck with the following parameters: gross vehicle weight m_v = 12,000 kg; kinematic parameters of the planetary gear sets of the speed gearbox $k_1 = k_2 = 2.74$; gear ratio of the single-reduction gear $i = 2$; gear ratios of the gearbox $u_1 = 3.74$, $u_2 = 2.16$; gear ratio of the main gear $u_{main} = 3.2$; coefficient of rolling resistance $f = 0.01$; wheel radius r_w = 0.392 m; frontal area of the vehicle F_v = 7.94 m^2; coefficient of air resistance k_a = 0.6 N·s^2/m^4; moment of inertia of the electric motor rotor I_m = 0.5 kg·m^2; reduced moment of inertia of the translationally moving mass and wheels of the electric vehicle I_v = 181.3 kg·m^2; rated torque of the electric motor M_{EMn} = 414 N·m; friction clutch safety factor with static coefficient of friction $\beta = 2$; efficiency of power unit gear trains $\eta_i = \eta_1 = \eta_2 = \eta_{main} = 0.98$.

From the acceleration graph of the electric vehicle (see Figure 2), it can be seen that when accelerating in the 2nd gear, the acceleration is greater than in the 1st. Therefore, it is advisable to shift gears from 1st to 2nd at a speed $V_1 = V_{02}$ (see Figure 1). The value of this speed is calculated by the formula

$$V_1 = \frac{n_0 r_w}{9.55 i u_2 u_{main}} = \frac{3{,}000 \cdot 0.392}{9.55 \cdot 2 \cdot 2.16 \cdot 3.2} = 8.91 \text{ m/s},$$

where n_0 = 3,000 rpm is the rated motor speed.

The moment of resistance to movement is calculated by the expression (5) with α = 0

$$M_r = \frac{\left[0.01 \cdot 12{,}000 \cdot 9.81 + 0.6 \cdot 7.94 \cdot 8.91^2\right] \cdot 0.392}{3.2 \cdot 0.98} = 194.4 \text{ N} \cdot \text{m}.$$

The coefficient k_μ is found by the formulas (1), (2) for β = 2

$$k_\mu = \frac{M_{f\,max}}{t_M} = \frac{414 \cdot 2 \cdot 2 \cdot 2.74 \cdot 0.98^2}{0.6} = 7{,}263,$$

where t_M = 0.6 s is the duration of the friction moment increase in the clutch.

The optimal overlap time is determined from (6)

$$t_1 = \frac{2.74 \cdot 2.74}{1 + 2.74 + 2.74} \times \frac{\left[0.01 \cdot 12{,}000 \cdot 9.81 + 0.6 \cdot 7.94 \cdot 8.91^2\right] \cdot 0.392}{2.16 \cdot 7{,}263 \cdot 3.2 \cdot 0.98} = 0.0144 \text{ s}.$$

The value of the electric motor torque is obtained from the formula (15) with $t = t_1$

$$M_m(t_1) = \frac{194.4}{2 \cdot 2.16 \cdot 0.98^2} = 46.9 \text{ N} \cdot \text{m}.$$

The calculation by formulas (8) gives the values of the parameters α_1 = 0.8631, α_2 = 1.8631.

After substituting the initial and calculated above parameters into the equation (17), an equation is obtained from which the slipping time $t = t_s$ of the crown gear of the 1st gear set of the gearbox is found:

$$\left(1647.6 - 0.5k_m\right)t^2 - \left(49.21 - 0.0144k_m\right)t - \left(114.6 + 1.04 \cdot 10^{-4} k_m\right) = 0. \quad (18)$$

As can be seen from the equation (18), the slipping time depends on the rate of increase of the electric motor torque k_m. Table 1 shows the results of calculating the t_s value for various values of the coefficient k_m, as well as the values of the friction moment $M_{f2}(t_s)$ in the friction clutch FC_2 calculated by the formula (1).

For the value k_m = 500 N·m/s, Table 2 shows the values of the motor shaft rotation speed $n_m = 9.55\omega_m$, the motor torque M_m, the speed of the electric vehicle V during the time interval (0, t_s), calculated by the formulas (9), (15), (16).

Figure 6 shows a graph of the change in the electric motor torque depending on the speed of the shaft over a period of time (0, t_s).

Multi-mass dynamic scheme (numerical calculation). The analytical calculation method discussed above based on the two-mass dynamic model of the electric vehicle power unit makes it possible to obtain a qualitative representation and a quantitative assessment of the gearshift process characteristics. Since this simplified vehicle model does not take into account the elastic properties of the transmission, for a more accurate description of the oscillatory dynamic processes occurring in it, it is necessary to consider a multi-mass dynamic model containing elastic elements.

Table 1 — Calculated values of the slipping time t_s of the 1st set crown gear of the gearbox

k_m, N·m/s	0	100	300	500
t_s, s	0.279	0.283	0.292	0.302
M_{f2}, N·m	2,026	2,055	2,121	2,193

Table 2 — Changing in values of the rotation speed of the electric motor shaft n_m, the electric motor torque M_m, the electric vehicle speed V during the 1st set crown gear slipping

t, s	0	0.0144	0.2	0.302
n_m, rpm	5,193	5,193	4,335	3,125
M_m, N·m	27.1	46.9	139.7	190.7
V, m/s	8.91	8.91	9.06	9.28

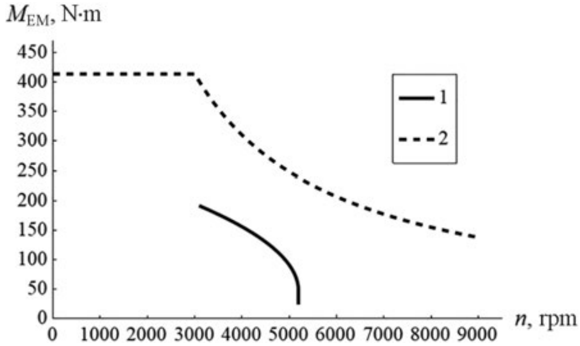

Figure 6 — Graph of the change in the torque of the electric motor depending on the speed of rotation of the shaft during the slipping time of the 1st set gear (curve 1), the external characteristic of the electric motor (curve 2)

Figure 7 shows a dynamic scheme of an electric vehicle power unit consisting of six masses with moments of inertia I_1–I_6; two rigid differentials D_1 and D_2 connecting masses I_1, I_2, I_3 and I_1, I_3, I_4, respectively; two elastic shafts with compliances E_{45} and E_{56}; two friction clutches FC_1, FC_2.

The process of changing the kinematic gear ratio of the gearbox from u_1 to u_2 over a period of time (t_1, t_s) is described by the following system of first order differential equations in the normal Cauchy form:

$$\begin{cases} \dot{\omega}_1 = \frac{1}{I_1}\left(M_m - M_1 - M_2\right), \\ \dot{\omega}_2 = \frac{1}{I_2}\left(-k_1 M_1 + M_{f2}\right), \\ \dot{\omega}_3 = \frac{1}{I_3}\left(\left(1 + k_1\right)M_1 - k_2 M_2 + M_{f1}\right), \\ \dot{\omega}_4 = \frac{1}{I_4}\left(\left(1 + k_2\right)M_2 - M_{45}\right), \\ \dot{\omega}_5 = \frac{1}{I_5}\left(M_{45} - M_{56}\right), \ \dot{\omega}_6 = \frac{1}{I_6}\left(M_{56} - M_r\right), \\ \dot{M}_{45} = \frac{\omega_4 - \omega_5}{E_{45}}, \ \dot{M}_{56} = \frac{\omega_5 - \omega_6}{E_{56}}, \end{cases} \quad (19)$$

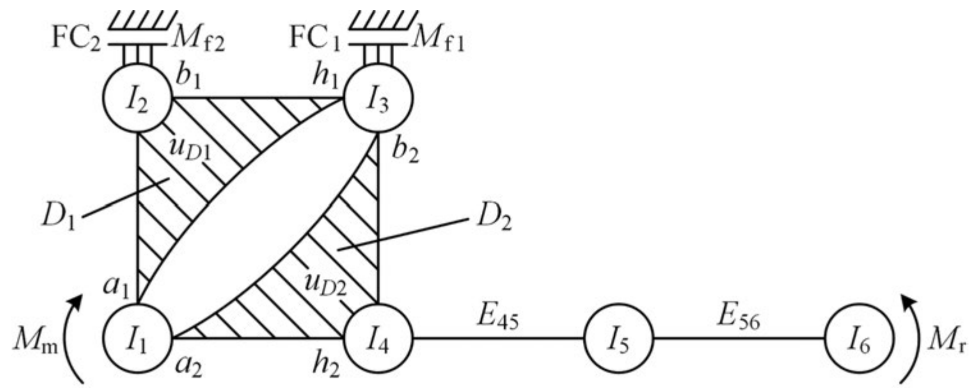

Figure 7 — Six-mass dynamic scheme of the electric vehicle power unit with the two-speed shaft-planetary gearbox: I_1 — moment of inertia of the electric motor rotor, the single reduction gear and the input shaft of the gearbox, reduced to the input shaft; I_2 — moment of inertia of the 1st planetary set crown gear with the driving discs of the clutch FC_2; I_3 — moment of inertia of the 1st planetary set carrier with satellites and the 2nd planetary set crown gear with the driving discs of the clutch FC_1; I_4 — moment of inertia of the 2nd planetary set gear carrier with satellites and the output shaft of the gearbox with a flange; I_5 — moment of inertia of the car driving wheels, reduced to the output shaft; I_6 — moment of inertia of the translationally moving mass and the car driven wheels, reduced to the output shaft; E_{45} — torsional compliance of the cardan shaft and axle shafts, reduced to the output shaft; E_{56} — torsional compliance of the wheel tires of the driving axle, reduced to the output shaft; $u_{D1} = -k_1 = -z_{b1}/z_{a1}$, $u_{D2} = -k_2 = -z_{b2}/z_{a2}$ — internal gear ratios of differentials D_1 and D_2 [10]; M_m — torque of the electric motor, reduced to the input shaft; M_r — moment of resistance to the movement of the electric vehicle, reduced to the output shaft; M_{f1}, M_{f2} — moments of friction in the friction clutches FC_1, FC_2

Table 3 — Parameters of the six-mass dynamic scheme: moments of inertia I_j, kg·m², torsional compliances E_{ij}, rad/(N·m)

I_1	I_2	I_3	I_4	I_5	I_6	E_{45}	E_{56}
2.022	0.0507	0.0618	0.045	0.586	180.7	$1.135\cdot10^{-4}$	$1.485\cdot10^{-5}$

where ω_j is the angular velocity of the j-th mass; M_1, M_2 are the internal moments acting in the first units a_1 and a_2 of the rigid differentials D_1 and D_2 [10]; M_{45}, M_{56} are the elastic moments in shafts with compliances E_{45} and E_{56}. The external moments acting on the system $M_m(t)$, $M_{f2}(t)$, $M_r(V)$ are determined respectively by the expressions (15) (taking into account the reduction shaft), (1) and (5) (with $V = \omega_6 r_w / u_{main}$); the moment $M_{f1}(t)$ is assumed to be zero.

Numerical integration of differential equations (19) was carried out by the Runge–Kutta method of the 4th order in the Mathematica computer algebra system. The calculated values of the parameters of the dynamic scheme are shown in Table 3. The initial conditions at time t_1 were set based on the assumption of uniform mass rotation corresponding to the movement of the vehicle at a speed $V_1 = V_{02}$ (see Figure 1), and the torsion of elastic shafts with moments equal in value $M_r(V_1)$.

Here are the comparative results of analytical calculations for the two-mass dynamic model (see Figure 3) and numerical calculations for the six-mass dynamic scheme (see Figure 7).

Figure 8 shows graphs of the change in the angular velocity of the crown gear b_1, defined by the expression $\omega_{b1}(t) = \alpha_2\omega_a(t) - \alpha_1\omega_{a1}(t)$, and the angular velocity $\omega_2(t)$ of mass I_2. Figure 9 shows graphs of changes in the angular velocities of the electric motor rotor $\omega_{a1}(t)$, the translationally moving mass of the electric vehicle $\omega_a(t)$ and the corresponding masses I_1 and I_6 – $\omega_1(t)$ and $\omega_6(t)$. Figure 10 shows graphs of the change in torque on the output shaft of the gearbox $M_a(t) = \alpha_2 M_{f2}(t)$ and elastic moments $M_{45}(t)$, $M_{56}(t)$ of shafts with compliances E_{45} and E_{56}.

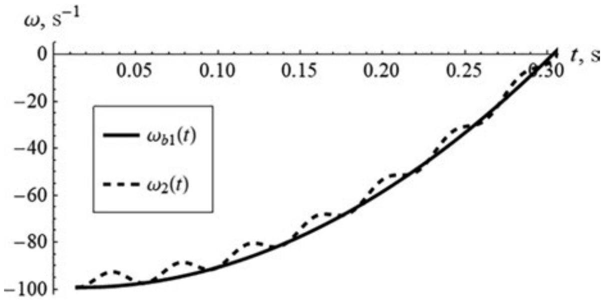

Figure 8 — Angular slipping speed of the crown gear b_1

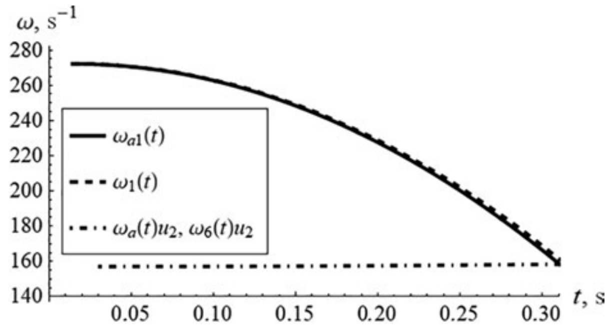

Figure 9 — Angular velocities of the electric motor rotor and the translationally moving mass of the electric vehicle, reduced to the gearbox input shaft

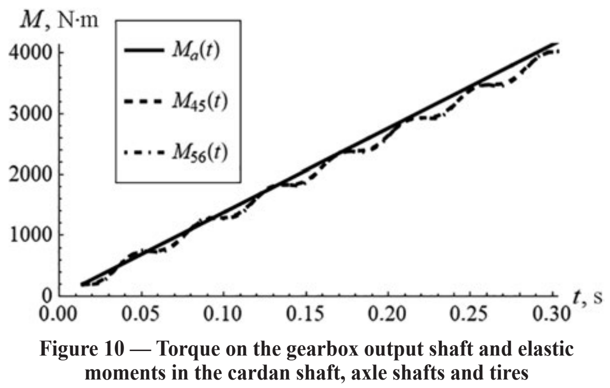

Figure 10 — Torque on the gearbox output shaft and elastic moments in the cardan shaft, axle shafts and tires

The oscillatory nature of the change in angular velocities and moments of the six-mass dynamic scheme is associated with the presence of elastic links in it. The introduction of additional masses I_2–I_4 is necessary to obtain the equations of motion of the elements of the gearbox planetary gear sets, and the allocation of a separate mass I_5 — to determine the elastic moments in the transmission. The moment of resistance M_r is assumed to be variable depending on the speed of the vehicle. Thus, it can be concluded that the results of calculations performed for two-mass and six-mass dynamic models are in good agreement.

The analytical calculation method using a two-mass dynamic model makes it possible to obtain a fairly accurate estimate of the characteristics of the gearshift process (overlap time, slipping time of the engaged friction clutch, change in the angular velocity of the motor shaft and the speed of the electric vehicle in time), to select the parameters of the laws of the motor torque change and friction moments in the friction clutches. These characteristics are expressed in the form of dependencies on the design and energy parameters of the electric vehicle, its driving conditions. Numerical calculations for the six-mass dynamic scheme make it possible to clarify the laws of change in speed and power factors, taking into account the oscillatory processes taking place, to determine the load indicators of the transmission and friction clutches, to evaluate and adjust the operation of the power unit control system in transient driving modes.

Conclusion. A method is proposed for calculating the load of electric vehicle gearboxes using dynamic models of the electric vehicle power unit, on the basis of which analytical dependences are obtained by means of Lagrange equations of the second kind to determine the slipping time of the friction clutches of the shaft-planetary gearbox.

The obtained analytical dependences of the gearshift process make it possible to optimize the control processes of friction clutches, thereby reducing dynamic loads in electric vehicle power units and increasing their energy efficiency.

Shifting gears from low to high should occur without interrupting the power flow with minimal energy loss for slipping clutches and minimal dynamic loads.

The analytical calculation method based on the two-mass dynamic model of the electric vehicle power unit makes it possible to get a qualitative representation and a quantitative estimate of the characteristics of the gearshift process (overlap time, slipping time of the engaged friction clutch, change in the angular velocity of the motor shaft and the speed of the electric vehicle in time), select the parameters of the laws of change of the engine torque and friction moments in the friction clutches. These characteristics are expressed in the form of dependencies on the design and energy parameters of the electric vehicle, its driving conditions. An example of calculating the gearshift time from low to high is given for the MAZ-4381 delivery electric truck.

For a more accurate description of vibrational dynamic processes in the transmission, taking into account its elastic properties, a multi-mass dynamic scheme is considered. Numerical calculations for the six-mass dynamic scheme made it possible to clarify the laws of change of velocity and force factors, taking into account the oscillatory processes taking place.

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МЕТОДЫ РАСЧЕТА НАГРУЖЕННОСТИ КОРОБОК ПЕРЕДАЧ ЭЛЕКТРОМОБИЛЕЙ С ИСПОЛЬЗОВАНИЕМ ИХ ДИНАМИЧЕСКИХ МОДЕЛЕЙ

Рассматривается двухмассовая динамическая модель электромобиля для получения аналитических зависимостей, описывающих процесс переключения передач в вально-планетарной коробке передач силового электропривода. В основу расчетов положена оптимальная диаграмма переключения передач с низшей на высшую без разрыва потока мощности от электродвигателя. На этапе переключения передач, на котором происходит изменение кинематического передаточного числа коробки передач от u_1 до u_2, для определения зависимостей от времени угловых скоростей входного и выходного валов коробки передач используются уравнения Лагранжа 2-го рода. Приведен пример расчета времени переключения передач с низшей на высшую для развозного электрогрузовика МАЗ-4381ЕЕ, а также выполнен уточненный расчет для многомассовой динамической модели.

Ключевые слова: *грузовой электромобиль, коробка передач, электродвигатель, диаграмма переключения передач, циркуляция мощности, динамическая модель, фрикционная муфта, уравнения Лагранжа 2-го рода, время буксования*

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