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NUMERICAL ANALYSIS OF DYNAMIC PROCESSES IN MULTISPEED PLANETARY TRANSMISSIONS WITH SPLITTING AND CIRCULATING POWER FLOWS

The article proposes an approach to synthesize mathematical models of multispeed planetary transmissions intended for numerical analysis of their dynamic characteristics under operating conditions that feature splitting and circulation of power flows (including gear shifting processes). The approach makes use of elements with spring-damper properties to model both epicyclic gear sets and friction clutches. The resulting system of ordinary differential equations has invariant structure and provides adequate calculation of the torques exerted in the elements of the epicyclic gears and the friction clutches across the entire operating range of the transmission. To verify the modeling approach, numerical experiments were conducted simulating operation of a production planetary transmission. The simulation results are presented showing transient operating modes with the friction clutches slipping and the power flows splitting and circulating.

Keywords: *multispeed planetary transmissions, mathematical modeling, transient modes, power circulation, power splitting, gear shifting, friction clutches*

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Introduction. Transmissions comprising epicyclic gear sets have found a wide application in automotive vehicles of different types and categories. The decades of progress in the areas of transmission design and control systems have brought forth mass-produced planetary transmissions having up to 10 speeds [1–3]. Such transmissions are characterized by complex designs featuring several degrees of freedom, multiple friction clutches, and various connections between epicyclic gear sets. When considering power paths, besides single-flow modes, planetary transmissions can feature multiple flows resulting from power splitting and/or power circulation. Mathematical modeling is one of the main tools to design, study and enhance planetary transmissions; it is used for analyzing transmission kinematics and dynamics, designing control systems, and assessing performance parameters of vehicles equipped with such transmissions.

Analysis of the literature dedicated to designing and studying mechanical transmissions, including those of the planetary type, allows to highlight the main methods employed for their modeling and simulation. One of the common approaches is to represent the vehicle and the transmission by one or two lumped rotating masses [4, 5]. This technique is used for evaluating the basic performance parameters of the vehicle, such as fuel economy, the maximum speed and acceleration, and the time to reach specific target speeds [4]. Models with higher complexity, comprising 3–5 masses or more, are used to analyze shifting processes [6, 7], loads and torsional oscillations in transmissions [8–11]. Besides lumped masses, these models include spring-damper elements. One can also find works that propose using the bond graph method as a formalism to derive matrices of lumped parameters [11–13].

When modeling planetary transmissions, it should be considered that in power circulation modes they become redundant systems, which cannot be modeled solely by inertial elements and gear ratios, thus requiring using spring-damper elements as they introduce additional degrees of freedom eliminating the redundancy. Moreover, for correct simulation of power splitting and power circulation modes, the model should calculate the actual torques exerted in all of the transmission branches including the elements of each epicyclic gear set and each friction clutch. This requirement rules out using models with equivalent (i. e. reduced) lumped parameters of inertia and stiffness, as these models calculate torques not "in situ" but rather in the locations of the lumped elements.

Another key problem in simulating planetary transmissions is how to model friction clutches. The prevailing approach employs models that approximate the coefficient of friction by a function of the slip speed and some additional variables such as friction plate temperature and pressure. The choice of the approximating method depends on the modeling tasks and usually ranges from the Coulomb model [7, 8] to the Karnopp model [11] and lookup tables populated with experimental data [14]. The major shortcoming of such approximations is a simplified representation of the "stick–slip" transition, i. e. in the form of switched discrete states. When the slip direction changes, it triggers an instantaneous switch in the friction torque sign, which, in turn, induces a torque pulse that does not align with the actual physics of the clutch and, thus, compels one to resort to ad hoc solutions to eliminate that adverse effect. In particular, in [14] the author introduces "necessary" and "exerted" friction torques and establishes transitions between these using a set of rules and "sensitivity" functions.

When using a friction approximation with discrete "stick–slip" transitions, one can synthesize the model of the transmission either in a form of several systems of differential equations each corresponding to a certain state of the clutches [9, 14], or as a single equation system with varying structure whose components can be switched on and off by activation functions that take values of 0 or 1 depending on the clutch states [8, 9]. The complexity of such models increases significantly with the number of gear ratios and the corresponding clutch operating combinations, due to either a considerable number of switched equation systems or a bulky structure of the varying system.

The above analysis shows the relevancy of elaborating an alternative approach to synthesize models of planetary transmissions. The approach should enable correct calculations of power splitting, power circulation, and torques in the elements of the epicyclic gear sets and the friction clutches, considering scenarios where the clutches may slip, adhere, or change slip directions. The model's structure should be relatively simple and remain invariant (i. e., constant) across all operating combinations of the clutches.

The approach to simulate the main elements of the transmission. To attain the formulated objective, one should adopt appropriate methods to model two types of transmission components, namely, epicyclic gear sets and friction clutches. To meet the above requirements regarding calculation of torques and power flows, the model of an epicyclic gear set should be a self-contained entity assigned with a set of kinematic, torque, and design properties. This can be implemented with an approach usually employed for load analysis of mechanisms with differential constraints. The approach implies using a partial system that characterizes an epicyclic gear set as a three-element differential mechanism (TDM) with the planet carrier having the spring-damper properties [8]. Because of the carrier's compliance, a speed discrepancy $\Delta\omega_{\mathrm{TDM}}$ is introduced into the TDM kinematics making the Willis equation[1] [16] take the following form:

$$\Delta\omega_{\mathrm{TDM}} = \frac{\omega_a + \omega_c k}{k+1} - \omega_b, \tag{1}$$

where k is the design parameter of the TDM equal to the ratio between the tooth numbers of the ring gear and the sun gear; $\omega_{...}$ are the angular speeds of the TDM elements. Here and below the variables associated with TDMs are assigned with indices a, b, and c designating the sun gear, the planet carrier, and the ring gear respectively.

In response to the speed discrepancy, the planet carrier exerts the torque defined by its spring-damper properties:

$$T_b = c_b \int \Delta\omega_{\mathrm{TDM}} dt + d_b \Delta\omega_{\mathrm{TDM}},$$

where c_b and d_b are the coefficients of torsional stiffness and damping. These can be identified using the equivalence methods for partial systems [8] and the transmission design data. When a frequency analysis of the transmission is not required, one can set these parameters to values providing minimum speed discrepancy $\Delta\omega_{\mathrm{TDM}}$ and full suppression of oscillations.

With the carrier torque known, the torques of the sun and the ring gears can be calculated as follows [16]:

$$T_a = \frac{-T_b}{(k+1)}; \quad T_c = \frac{-T_b k}{(k+1)}.$$

If the power losses are to be taken into account, one can introduce either an efficiency parameter or a drag torque into these expressions.

Using the kinematic discrepancy $\Delta\omega_{\mathrm{TDM}}$ in the equation (1) transforms the TDM model into a system with three degrees of freedom, which allows representing each element of the epicyclic gear set as a rotating mass whose inertia is the actual one defined by the transmission design. The motion of the elements, driven by the torques acting upon them, is described by differential equations of rotational dynamics. In that way, each epicyclic gear set of the transmission is simulated by a self-contained model block having three kinematic inputs (i. e.

[1] This work uses the equations of speeds and torques for an epicyclic gear set having a negative speed ratio (when the planet carrier is stopped) and consisting of the sun gear, ring gear, and the carrier with simple planet pinions.

the angular velocities of the TDM elements) and three torque outputs (the torques exerted in each of the TDM elements).

The model of friction used as the root element to simulate the transmission's clutches should adequately calculate "stick–slip" transitions and the inversion of slip direction. The frequently used clutch stick criterion based on reaching (or crossing) zero slip is, generally, incorrect [9], as the clutch will only adhere if the friction torque is sufficient to keep it engaged under the applied external loads. Otherwise, the clutch will not stick but rather change the direction of slippage. To reflect this in the model, one has to derive the conditions of "stick–slip" transition from a dedicated analysis of torque balance [9], which is not a trivial task, especially for transient processes and power circulation.

To eliminate the described problem, one can use a friction model based on the spring-damper principle. Its advantage is in "automatic" calculation of the torque (with no need of a dedicated analysis) as a function of kinematic variables whose values correspond to the loads applied at the both ends of the spring-damper element. Increasing the loads enlarges the kinematic difference between the endpoints of the element, which, in turn, increases its torque. If the element has a limited torque capacity, reaching its maximum will result in intensive growth of the kinematic difference, indicating clutch slip. Below the torque limit, the kinematic difference is low, which can be considered as microslip or sticking. When external loads exceed the torque limit of the friction link, slipping does not stop after reaching zero but continues in an opposite direction. A spring-damping link also allows for adequate simulation when the friction element is a part of a power circulation path. The similar principles underlie such models of friction as the Dahl model [17] and the LuGre model [17, 18] named after its authoring universities of Lund and Grenoble.

The LuGre model that was considered suitable for this work has been adapted for its purposes by replacing the friction force (as in the original model [18]) with the friction coefficient calculated for rotational motion:

$$\dot{z} = \omega_{\text{slip}} - \sigma_0 \frac{|\omega_{\text{slip}}|}{\mu_{\text{f,s}}(\omega_{\text{slip}})} z; \quad \mu_f = \sigma_0 z + \sigma_1 \dot{z},$$

where ω_{slip} is the angular slip speed; z is the friction state variable; σ_0 and σ_1 are the stiffness and damping coefficients of the friction link (per unit normal load), $\mu_{\text{f,s}}(\omega_{\text{slip}})$ is a functional relation between the steady slip speed and the friction coefficient; μ_f is the actual friction coefficient (including transient slip modes). Using $\mu_{\text{f,s}}(\omega_{\text{slip}})$ introduces a saturation effect preventing the actual (transient) friction coefficient from exceeding the steady-state values.

In the article [18], it is shown that the LuGre model makes adequate calculations of "stick–slip" transitions including the accompanying hysteresis effect. The model accuracy deteriorates for some peculiarities of "stick–slip" transients; however, such details make interest mostly for in-depth studies of the friction physics. In the modeling tasks that prioritize more general friction properties, these issues have low relevancy.

Usually, planetary transmissions feature wet multidisc clutches whose friction torque is calculated as follows:

$$M_{\text{f}} = F_{\text{N}} \cdot \mu_{\text{f}} \cdot R_{\text{f}} \cdot n_{\text{f}},$$

where F_{N} is the clamping (normal) force; R_{f} is the effective friction radius; n_{f} is the number of the friction plates. The drag torque can be taken into account by an additional term.

A multidisc clutch is typically operated by an electrohydraulic control system. The combination of a hydraulic cylinder, a piston, and a return spring is referred to as a booster. Assuming that the centrifugal pressure within the booster is fully cancelled by means of the compensation chamber, and neglecting the dynamics of the piston mass as well as its contact friction with the cylinder wall, one can calculate the clamping force from the following equilibrium equation:

$$F_{\text{N}} = pA_{\text{p}} - F_{\text{spr}},$$

where p is the control pressure; A_{p} is the piston working area; F_{spr} is the force exerted by the return spring.

When using the described approaches to synthesize the models of the TDMs and the friction elements, one obtains a system of ordinary differential equations whose structure remains unaffected by the states of the clutches or the configuration of the epicyclic gear sets (i. e. how their elements are currently connected to one another or to the transmission casing). Thus, all the states of the transmission, gearshifts included, are modeled by a single invariant equation system, which is capable of calculating the actual torques in all of the transmission branches including those involved in power splitting and circulation paths. Note that this system is mostly composed of stiff differential equations, requiring the use of an implicit solver and a sufficiently small time step for numerical integration. However, the rapidity of gear shifting processes also implies using small time steps making that requirement more a necessity than a drawback.

The studied transmission and its mathematical model. To verify and validate the proposed modeling approach, a numerical study was conducted simulating a 9-speed production planetary transmission intended for passenger vehicles [19]. Its schematic is shown in Figure 1. The main components of the transmission are four epicyclic gear sets (designated as EG1–EG4) and six friction clutches (designated as FC1–FC6).

In the transmission model, the components of the epicyclic gear sets are represented by the iner-

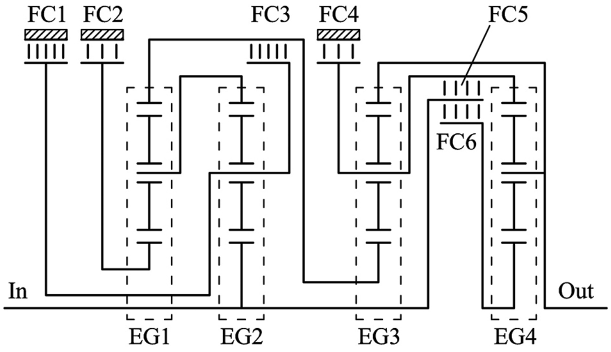

Figure 1 — Schematic of the studied planetary transmission [19]

tial elements $J_{...}$ whose rotations, driven by the torques $T_{...}$, are described by the following differential equations:

$$\begin{cases} J_{\text{in}}\dot{\omega}_{\text{in}} = T_{\text{in}} + T_{a2} - T_{\text{FC5}} - T_{\text{FC6}}, \\ J_{b2}\dot{\omega}_{b2} = T_{b2} + T_{\text{FC1}} - T_{\text{FC3}}, \\ J_{b1c2}\dot{\omega}_{b1c2} = T_{b1} + T_{c2}, \\ J_{a1}\dot{\omega}_{a1} = T_{a1} + T_{\text{FC2}}, \\ J_{c1a3}\dot{\omega}_{c1a3} = T_{c1} + T_{a3} + T_{\text{FC3}}, \\ J_{b3c4}\dot{\omega}_{b3a4} = T_{b3} + T_{c4} + T_{\text{FC4}} + T_{\text{FC5}}, \\ J_{a4}\dot{\omega}_{a4} = T_{a4} + T_{\text{FC6}}, \\ J_{c3b4}\dot{\omega}_{c3b4} = T_{c3} + T_{b4} - \sum_{i=1}^{2} T_{\text{shaft},i}. \end{cases} \qquad (2)$$

The numerical indices of the variables correspond to the numbers of the EGs and the FCs in Figure 1. The elements shared between two EGs are assigned with double indices; for instance, b_1c_2 stands for the element incorporating the planet carrier of EG1 and the ring gear of EG2. The variables associated with the transmission input shaft have “in” in their indices.

In the model, the torques at the epicyclic gears and the clutches were calculated in accordance with the approaches described in the previous section without taking into account meshing and bearing losses (as they had no relevance for the study at hand). In the friction model, the $\mu_{\text{f,s}}(\omega_{\text{slip}})$ function was derived from physical tests of a “paper-to-steel” wet contact pair whose maximum friction coefficient remained approximately 0.125 across most of the operating slip range.

The vehicle considered in the study has a transfer case connected to the output of the planetary transmission to distribute the torque between the shafts propelling the front and the rear wheels. In the equations (2), the loading torques transmitted by these shafts to the output of the planetary transmission are denoted $T_{\text{shaft},i}$, where i is the index of the shaft (1 for the front one, and 2 for the rear).

To introduce the driving torque saturation imposed by tire adhesion, the model is augmented with the wheel dynamics using the following equations (one for each axle):

$$J_{\text{w}}\dot{\omega}_{\text{w},i} = T_{\text{shaft},i}u_0 - (R_{z,i}\mu_{\text{tire},i} + R_{z,i}f_{\text{roll},i})r_i,$$

where J_{w} is the inertia of the wheels (per axle); $\omega_{\text{w},i}$ is the wheel angular speed; u_0 is the final drive ratio; $R_{z,i}$ is the wheel normal force; r_i is the wheel radius; $\mu_{\text{tire},i}$ is the coefficient of tire adhesion; $f_{\text{roll},i}$ is the coefficient of rolling resistance; i is the axle index.

In the study, it is assumed that the vehicle moves linearly along a horizontal surface. With these assumptions, the equation of vehicle dynamics reads:

$$m_{\text{veh}}\dot{v}_{\text{veh}} = \sum_{i=1}^{2}\left(R_{z,i}\cdot\mu_{\text{tire},i}\right) - F_w,$$

where m_{veh} is the vehicle mass (3,500 kg in the simulations described below); v_{veh} is the vehicle velocity, and F_w is the air drag force.

In the simulations, the adhesion coefficient μ_{tire} was approximated by a non-linear function of the tire slip, which was calculated from the velocities of the vehicle v_{veh} and the wheels ω_{w} [20]. The normal forces R_z were derived from the equilibrium of the external moments applied to the vehicle [20]. Finally, the air drag force F_w was approximated by a quadratic function of the vehicle velocity [20].

Numerical analysis of dynamic processes in the planetary transmission. The conducted numerical experiment simulated the transmission operating in transient modes. To this effect, the simulation commenced with the vehicle taking off, followed by driving with the engine throttle set to its maximum. Figure 2 shows the variables describing the operation of the transmission during the take-off phase, namely, the hydraulic pressure in the engaging clutch FC1 (*a*), the friction coefficient (*b*) and the friction torque (*c*) of FC1, the rpms (*d*) of the transmission input and output shafts as well as the rpm of the rotating parts of FC1. To make the rpm graphs look more illustrative, the output rpms were “translated” to the input shaft by multiplying them with the ratio of the first gear (u_1), which was engaged during the take-off.

The graphs show that, for most of its duration, the engagement process of FC1 maintains the friction coefficient at its maximum value of 0.125. As the slip speed diminishes, the external loads decrease leading to a proportional reduction of the friction torque. FC1 engages completely after 1 second. Subsequently, the control pressure rises gradually up to its maximum of 20 bars causing a significant drop in the friction coefficient. In addition to using FC1, the first gear employs FC2 and FC4 clutches, both fully engaged. Power is transmitted as a single flow from the input shaft to EG2 and further to EG1, EG3, and EG4.

Figure 3 demonstrates the variables describing the operation of the transmission in modes featuring power circulation and splitting, namely, in the 4th and 5th gears and in shifting between these. The graphs show the vehicle velocity (*a*), the rpms of the input and output shafts (*b*), the powers at the transmission input and output (*c*), the control pressures of the clutches (*d*), and the powers at the elements of the epicyclic gear sets (*e*–*h*).

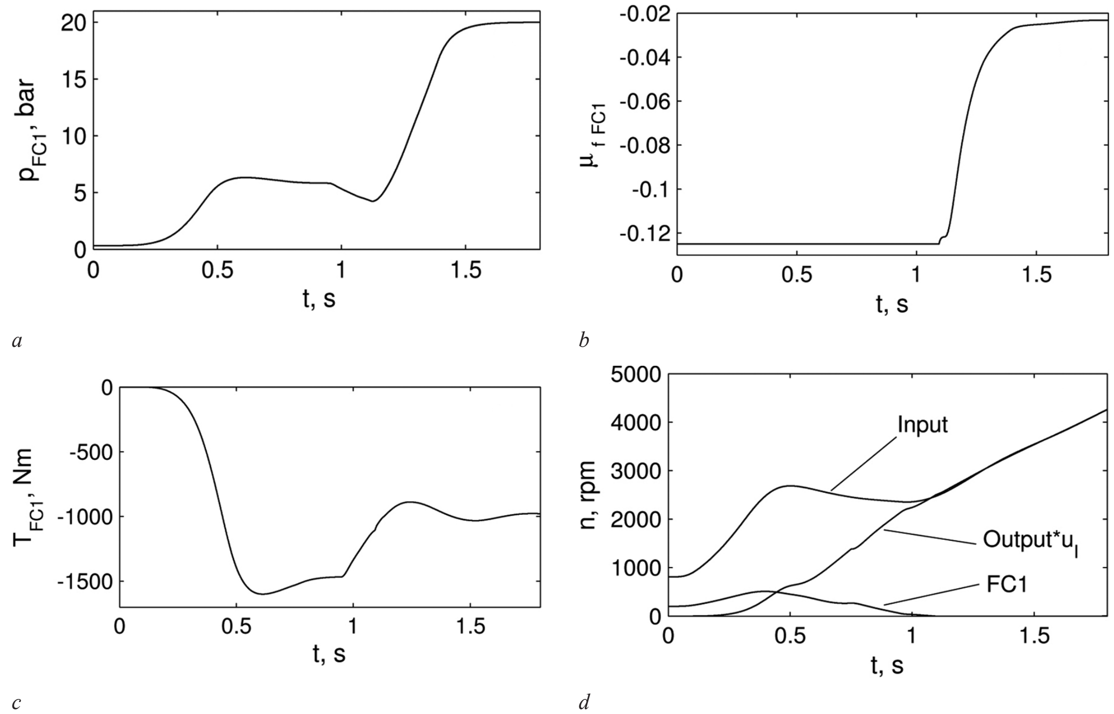

Figure 2 — Simulation results. Vehicle take-off

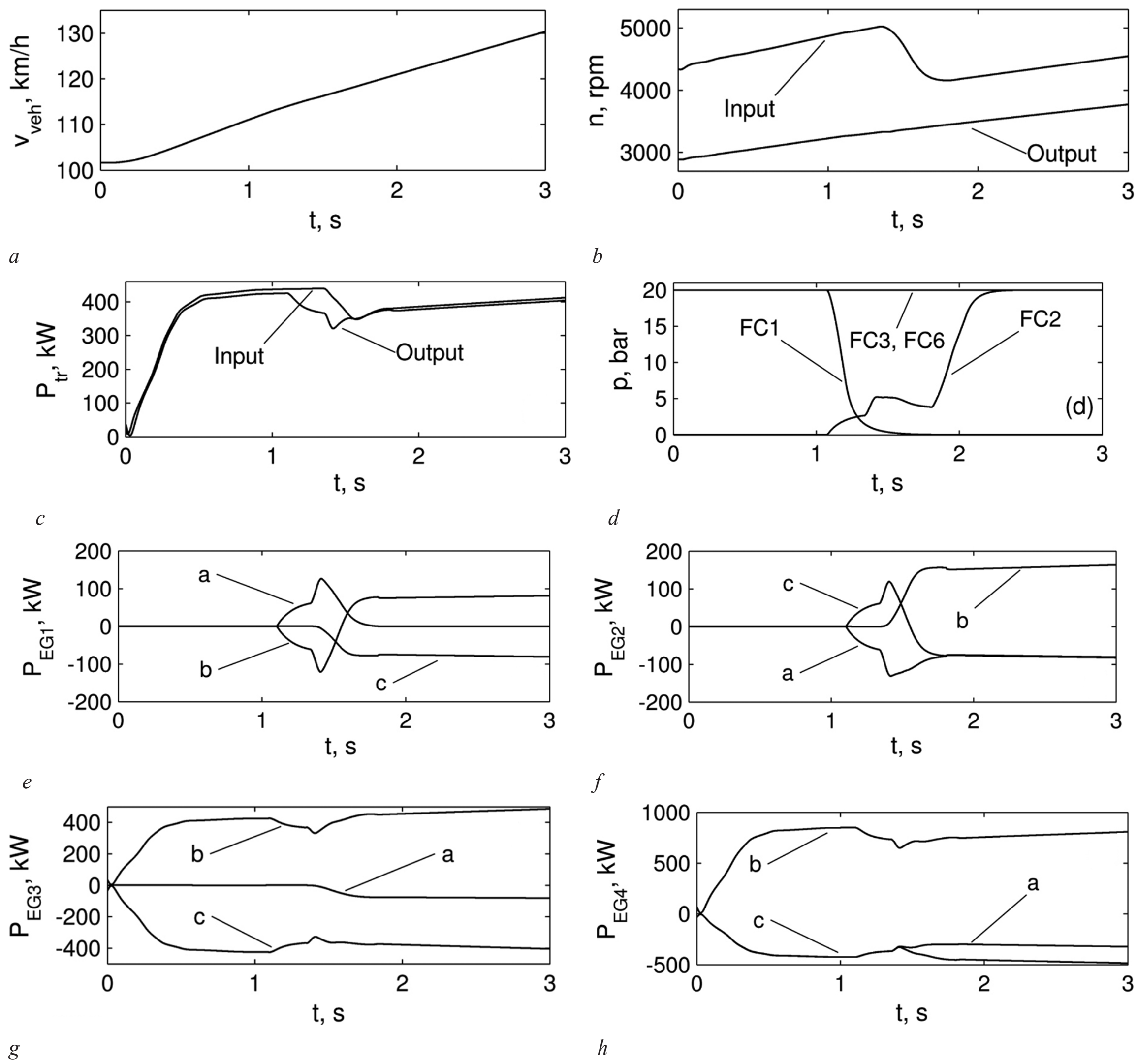

Figure 3 — Simulation results. Shifting between 4th and 5th gears

The gear shifting process consists of two main phases, namely, the torque phase and the inertia phase [2, 6, 7]. In the torque phase, one of the current gear's clutches gradually disengages while one of the next gear's clutches simultaneously engages. This seamless transition changes the torque ratio of the transmission without interrupting the power flow. In the inertia phase, the next gear's clutch engages completely thereby changing the kinematic ratio of the transmission. In the studied transmission, the 4th gear utilizes clutches FC1, FC3, and FC6, while the 5th gear employs FC2, FC3, and FC6. Thus, shifting between these gears involves overlapped switching of FC1 and FC2 clutches, which is clearly seen in the pressure graphs (see Figure 3 *d*). As these two clutches are connected to the elements of EG1 and EG2, their switching causes a rearrangement of the power flows through the corresponding gears evident in the graphs (see Figure 3 *e*, *f*) between 1 and 2 seconds. Specifically, the sun gear of EG1 ceases to transmit power, being braked by FC2, while the planet carrier of EG2 begins transmitting power as FC1 clutch becomes unbraked.

Prior to conducting the simulations, all the gear ratios of the studied transmission were identified by both calculations, using the design parameters of the epicyclic gear sets, and physical tests performed on a test bench and a vehicle. Specifically, the ratios of the 4th and 5th gears were identified as 1.511 and 1.205, respectively. The simulation results show the same ratio values (with the error of less than 1 %) calculated for both speeds and torques[2] using their magnitudes at the transmission input and output shafts. This consistency allows the model to be considered generally valid.

With the meshing and bearing losses neglected, the differences between the input and output powers (see Figure 3 *c*) arise from inertial effects and from power dissipation by the slipping clutches during the gearshift. Upon transitioning to the 5th gear, these powers become sufficiently close, allowing them to be treated as equal in the subsequent power flow analysis.

The simulation results were used to derive and visualize the power flows of the studied transmission in the 5th gear. This gear is of particular interest as it involves simultaneous power circulation, splitting, and merging. A visualization of the power flows is presented in Figure 4 showing the flow directions, the circulation loops, and the nodes of power splitting and merging. The numerical values of the powers in the transmission branches are also shown expressed as fractions of the input power. The magnitudes of these fractions allowed rounding them to the first decimal digit with almost no loss of precision.

The schematic reveals that in the 5th gear the transmission has two power circulation loops. Splitting of power flows occurs at the input and output shafts, and at the node where FC3 clutch connects with the element incorporating the ring gear of EG1 and the sun gear of EG3. 20 % of the input power proceeds to the sun gear of EG2 and then forms the circulation loop that passes through EG1 and EG2 gear sets. The remaining 80 % of the input power, along with the power exiting from the FC3 node of the first circulation loop, form the second circulation loop enveloping EG3 and EG4. The EG4 planet carrier, being a part of the second loop, transmits the circulating power twice as high as that at the input shaft. Merging of power flows occurs at three nodes coinciding with the planet carriers of EG2, EG3, and EG4.

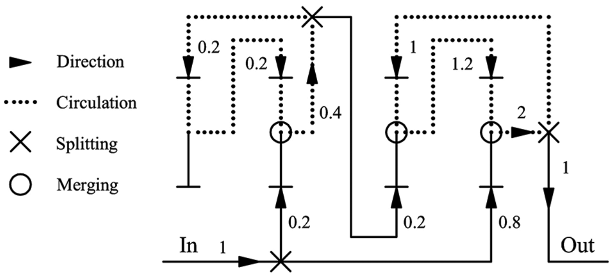

Figure 4 — Power flows within the planetary transmission in the 5th gear

Conclusions. The proposed approach allows synthesizing a mathematical model of a planetary transmission in the form of a system comprising ordinary differential equations, as well as speed and torque relations, whose structure is invariant for all of the transmission states (i. e. states of the friction clutches). Using the spring-damper principle to simulate both epicyclic gear sets and friction clutches eliminates the redundancy problem allowing for calculation of actual torques in the transmission elements including those involved in power circulation loops. The utilized friction model constitutes a continuous dynamic process, which allows simulating "stick–slip" transitions and the inversion of slip direction with no need of a dedicated torque analysis and avoiding instantaneous switching of clutch states.

The conducted simulation provided an insight into the operation of the planetary transmission in transient modes with the clutches slipping during the take-off and the gearshift. The power flow analysis, based on the simulation results, revealed the magnitudes of the powers passing through the transmission branches, as well as the power circulation loops and the nodes of power splitting and merging.

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ВЫЧИСЛИТЕЛЬНЫЙ АНАЛИЗ ДИНАМИЧЕСКИХ ПРОЦЕССОВ В МНОГОСТУПЕНЧАТЫХ ПЛАНЕТАРНЫХ ТРАНСМИССИЯХ С РАЗДЕЛЕНИЕМ И ЦИРКУЛЯЦИЕЙ ПОТОКОВ МОЩНОСТИ

В статье предложен подход к формированию математических моделей многоступенчатых планетарных трансмиссий, позволяющий численно исследовать происходящие в них динамические процессы в условиях разделения и циркуляции мощности, в том числе при переключениях передач. Подход основан на использовании упругодемпфирующих связей для моделирования как планетарных механизмов, так и фрикционных элементов. В результате формируется система обыкновенных *дифференциальных уравнений с постоянной структурой, которая адекватно отражает нагрузки, действующие на звенья планетарных механизмов и фрикционные элементы, во всем рабочем диапазоне трансмиссии. Работоспособность и адекватность полученной таким образом модели иллюстрируется посредством вычислительного анализа режимов работы планетарной трансмиссии серийного производства. Представлены показатели, характеризующие функционирование трансмиссии в неустановившихся режимах со скольжением фрикционных элементов, с циркуляцией и разделением потоков мощности.*

Ключевые слова: *многоступенчатые планетарные трансмиссии, математическое моделирование, переходные процессы, циркуляция мощности, разделение мощности, переключение передач, фрикционные элементы*

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